

# Passivity Based Control

applied to

# Dynamic Positioning of Ships

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# Outline

- Passivity Based Control using port-Hamiltonian formulation
  - Port-Hamiltonian modeling
  - The IDA-PBC technique
- Mathematical Model of Ship
- Dynamic Positioning problem
- Family of Passivity Based Controllers
  - Static IDA-PBC controllers
  - Dynamic IDA-PBC controllers
- Summary and Conclusions

# Passivity Based Control using port-Hamiltonian formulation

## Port- Hamiltonian Structure

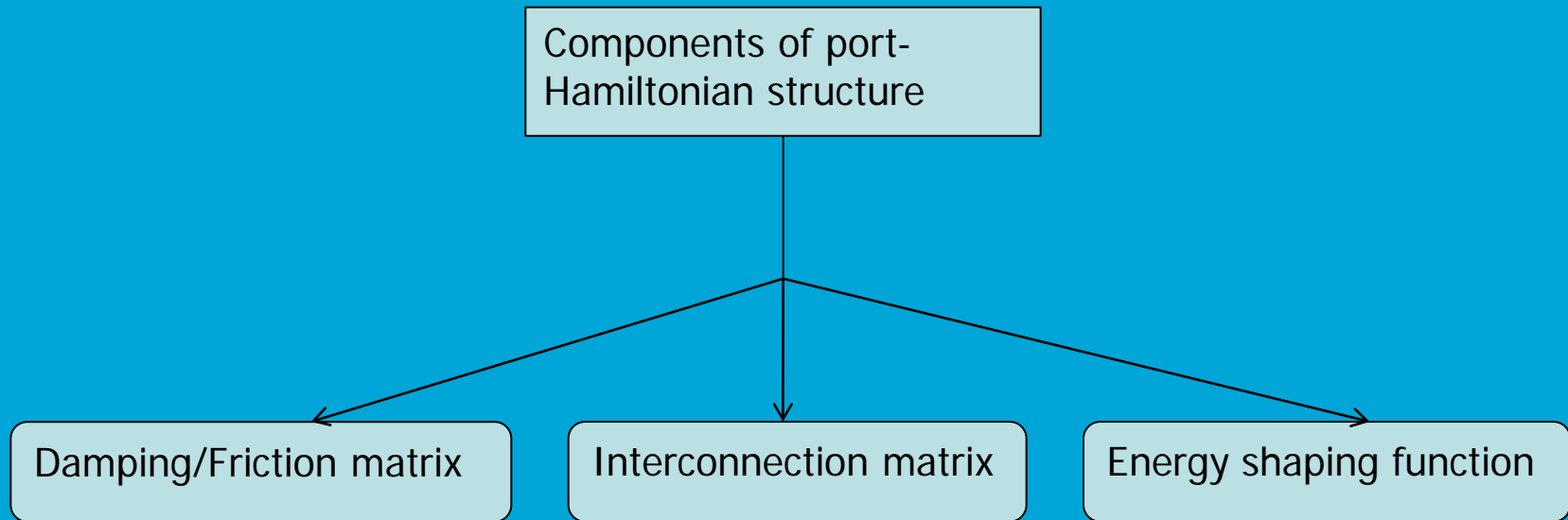


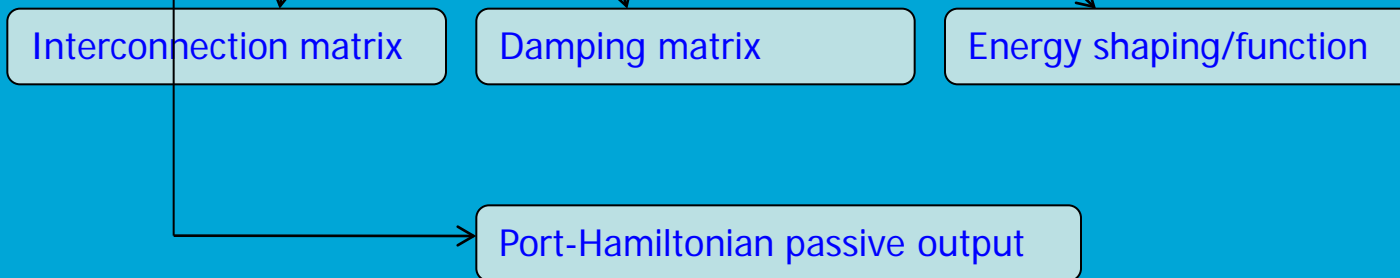
Figure 1. A description of port-Hamiltonian structure.

# Passivity Based Control using port-Hamiltonian formulation

## 1. Port-Hamiltonian model of a system

$$\dot{x} = (\mathfrak{S} - \mathfrak{R})\partial H(x) + g(x)u, \quad (1)$$

$$y = g^T(x)\partial H(x) \quad (2)$$



# Passivity Based Control using port-Hamiltonian formulation

## 2. IDA-PBC methodology

Desired closed loop dynamics

$$\dot{x} = (\mathfrak{I}_d - \mathfrak{R}_d) \partial H_d(x), \quad (3)$$

Equate (2) and (3), and solve for "u"

Control law

$$u = (g^T g)^{-1} g^T ((\mathfrak{I}_d - \mathfrak{R}_d) \partial H_d(x) - (\mathfrak{I} - \mathfrak{R}) \partial H), \quad (4)$$

# Mathematical Model of Ship

## 1. Model in Cartesian coordinates

$$\begin{bmatrix} \dot{\eta} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} O_3 & J(\psi) \\ O_3 & -M^{-1}D \end{bmatrix} \begin{bmatrix} \eta \\ v \end{bmatrix} + \begin{bmatrix} O_3 \\ M^{-1} \end{bmatrix} \tau + \begin{bmatrix} O_3 \\ M^{-1}J^T(\psi) \end{bmatrix} \quad (5)$$

$$\eta = [x \quad y \quad \psi]^T, \quad v = [u \quad v \quad r]^T$$

$$M = M^T = \begin{bmatrix} m_{11} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & m_{22} & m_{23} \\ \mathbf{0} & m_{32} & m_{33} \end{bmatrix} \geq \mathbf{0},$$

$$D = D^T = \begin{bmatrix} d_{11} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & d_{22} & d_{23} \\ \mathbf{0} & d_{32} & d_{33} \end{bmatrix} \geq \mathbf{0}$$

# Mathematical Model of Ship

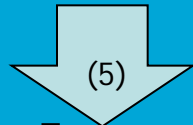
## 2. Model in port-Hamiltonian (energy) coordinates

Coordinate transformation

$$q = \eta, \quad p = Mv$$

Choice of energy shaping

$$H(q, p) = \frac{1}{2} q^T K q$$



$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{pmatrix} O_3 & J(q_3) \\ -J^T(q_3) & O_3 \end{pmatrix} - \begin{bmatrix} O_3 & O_3 \\ O_3 & D \end{bmatrix} \partial H + \begin{bmatrix} O_3 \\ I_3 \end{bmatrix} \tau + \begin{bmatrix} O_3 \\ J^T(q_3) \end{bmatrix} b \quad (6)$$

$$q = [q_1 \quad q_2 \quad q_3]^T, \quad p = [p_1 \quad p_2 \quad p_3]^T$$

# Dynamic Positioning Problem

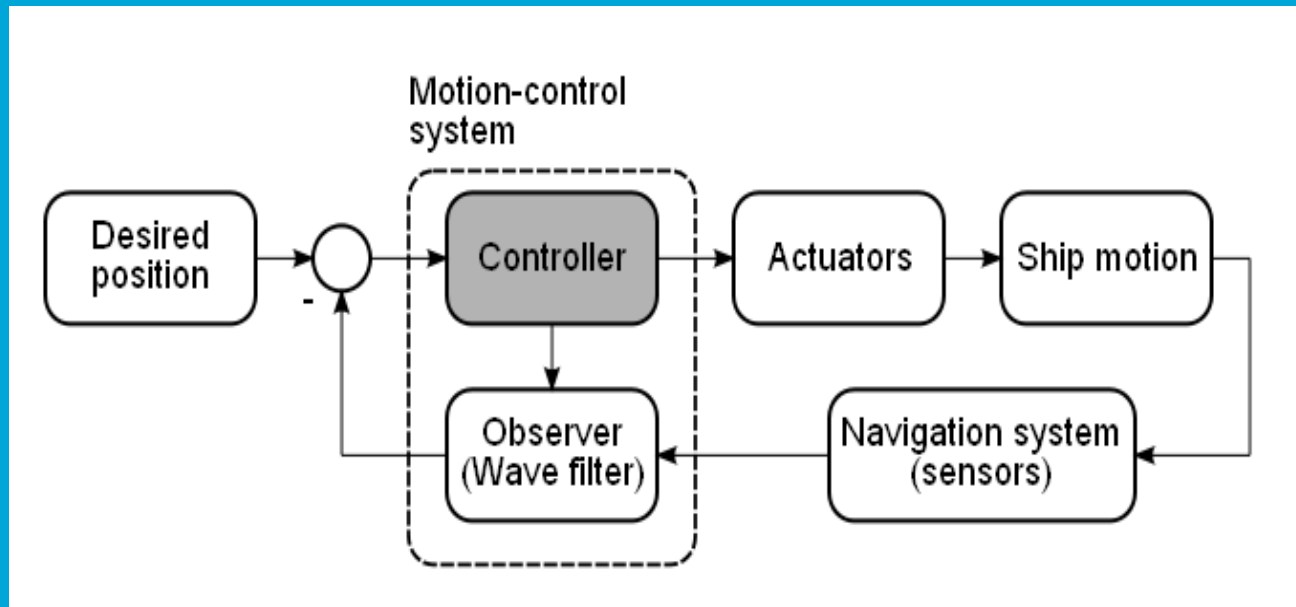


Figure 2. A description of motion control system.

# Dynamic Positioning Problem

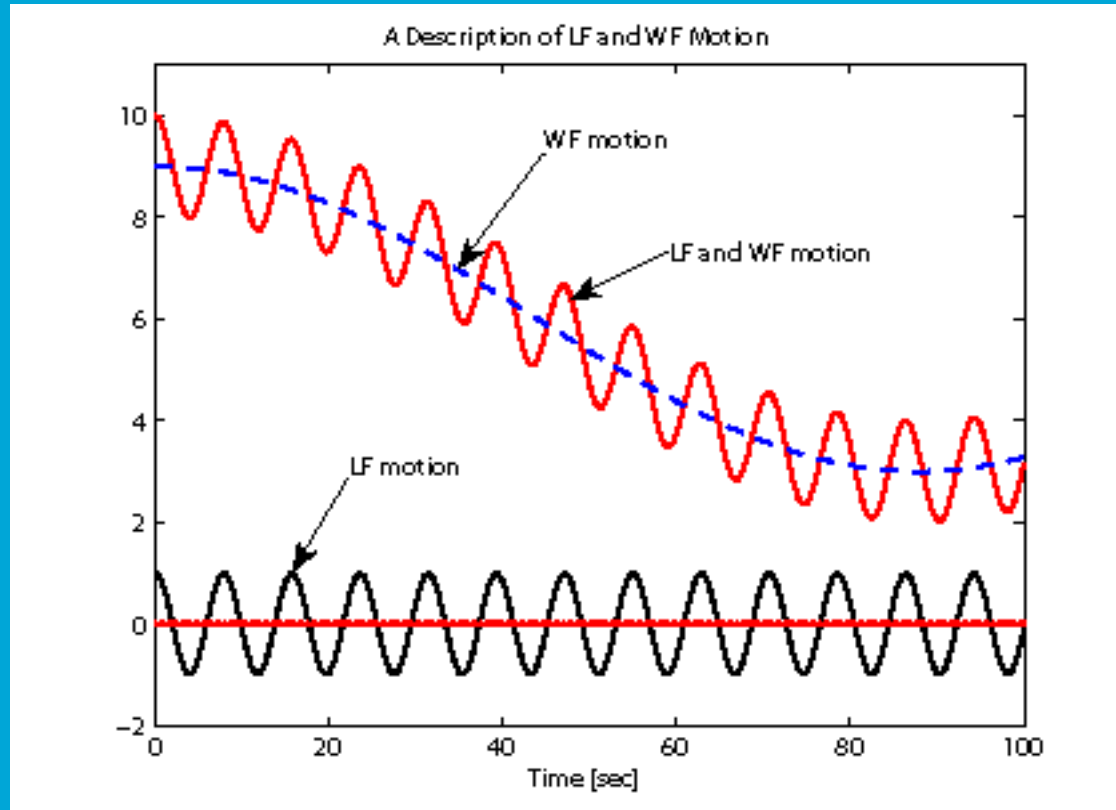


Figure 3. A description of LF and WF motions

# Quadratic vs Trigonometric energy shaping

Two different energy functions

$$\rightarrow H_{d1}(q, p) = \frac{1}{2} q^T K q + \frac{1}{2} p^T M^{-1} p \quad (7)$$

$$H_{d2}(q, p) = \frac{1}{2} q_{12}^T C_{12} q_{12} + c_3 (1 - \cos q_3) + \frac{1}{2} p^T M^{-1} p, \quad (8)$$

Quadratic  
energy shaping

Trigonometric  
energy shaping

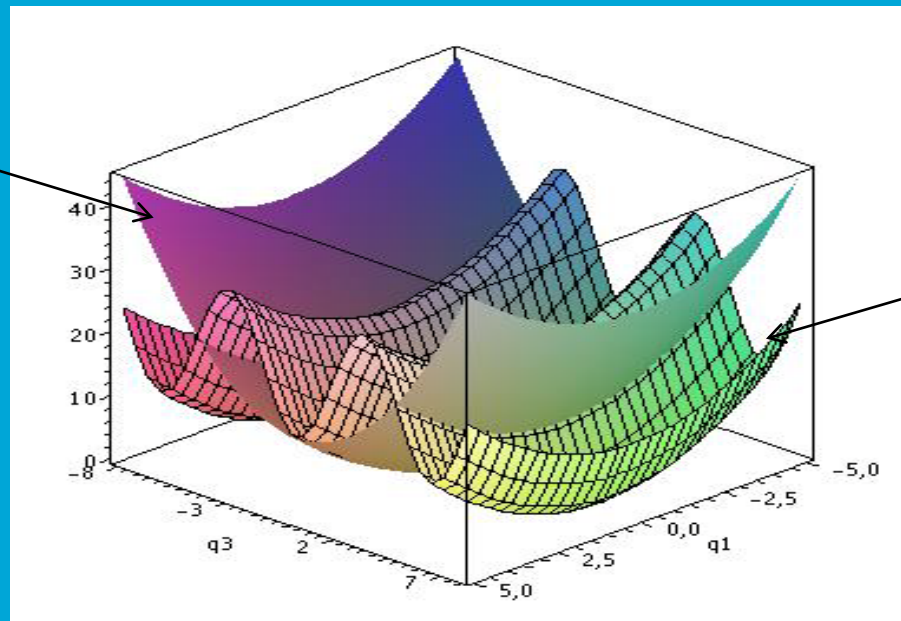


Figure 4. A description of quadratic and trigonometric energy shapings.

# Quadratic vs Trigonometric energy shaping

Effect on the performance of the controller and the choice of the control law

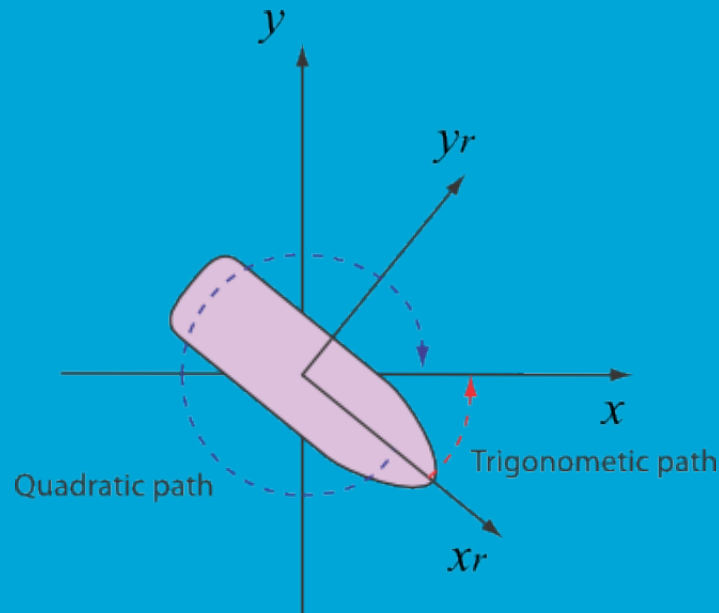


Figure 5. A description of the effect of the energy shaping on the heading control of the ship.

# Family of Passivity Based Controllers

## 1. Static quadratic controller

A nonlinear output feedback PD-controller with a feed forward term

$$H_{d1}(q, p) = \frac{1}{2} q^T K q + \frac{1}{2} p^T M^{-1} p$$

$$\tau_{sq} = -J^T(q_3)(Kq + b) - K_D J^T(q_3) \dot{q} \quad (9)$$

## 2. Static trigonometric controller

A nonlinear output feedback PD-controller with a feed forward term

$$H_{d2}(q, p) = \frac{1}{2} q_{12}^T C_{12} q_{12} + c_3(1 - \cos q_3) + \frac{1}{2} p^T M^{-1} p,$$

$$\tau_{st} = -J^T(q_3) \left( C \begin{bmatrix} q_1 \\ q_2 \\ \sin q_3 \end{bmatrix} + b \right) - K_D J^T(q_3) \dot{q}. \quad (10)$$

# Family of Passivity Based Controllers

- Quadratic vs trigonometric energy shaping

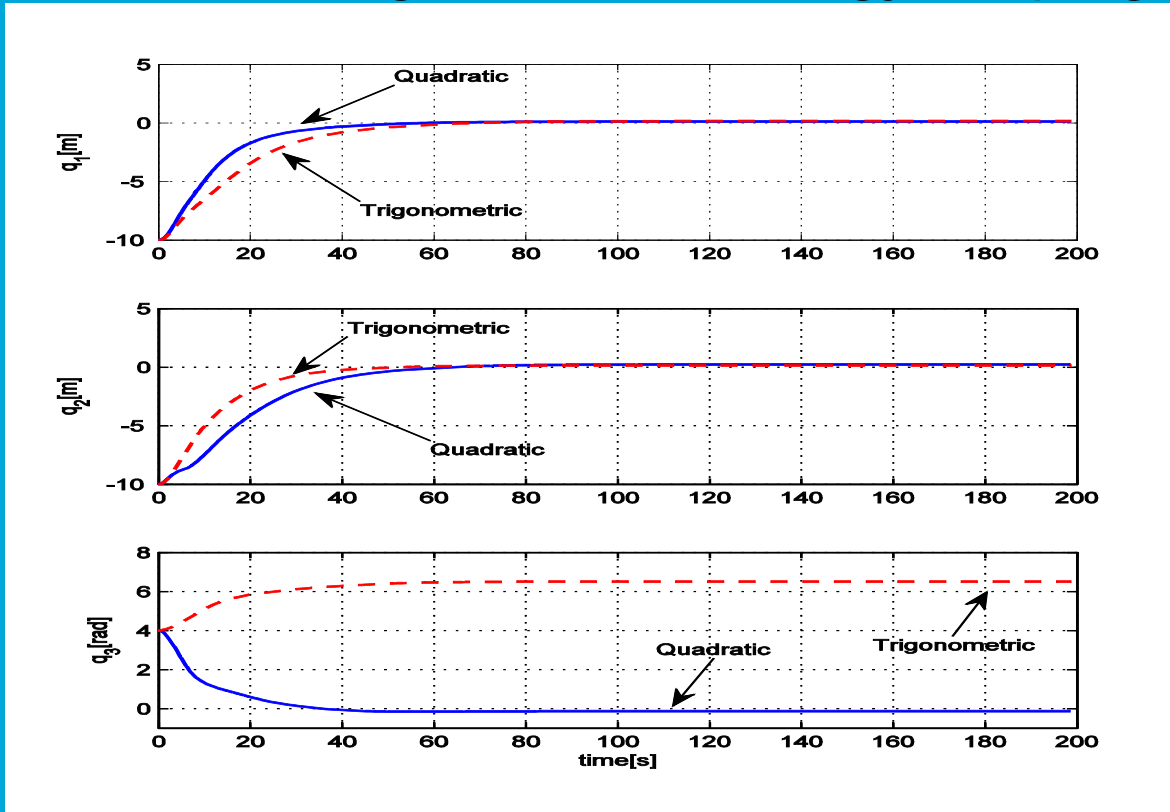


Figure 6. Ship position coordinates,  $q_1$  and  $q_2$ , and heading angle  $q_3$ , for the static quadratic controller (solid line) and the trigonometric controller (dashed line).

# Family of Passivity Based Controllers

- Quadratic vs trigonometric energy shaping

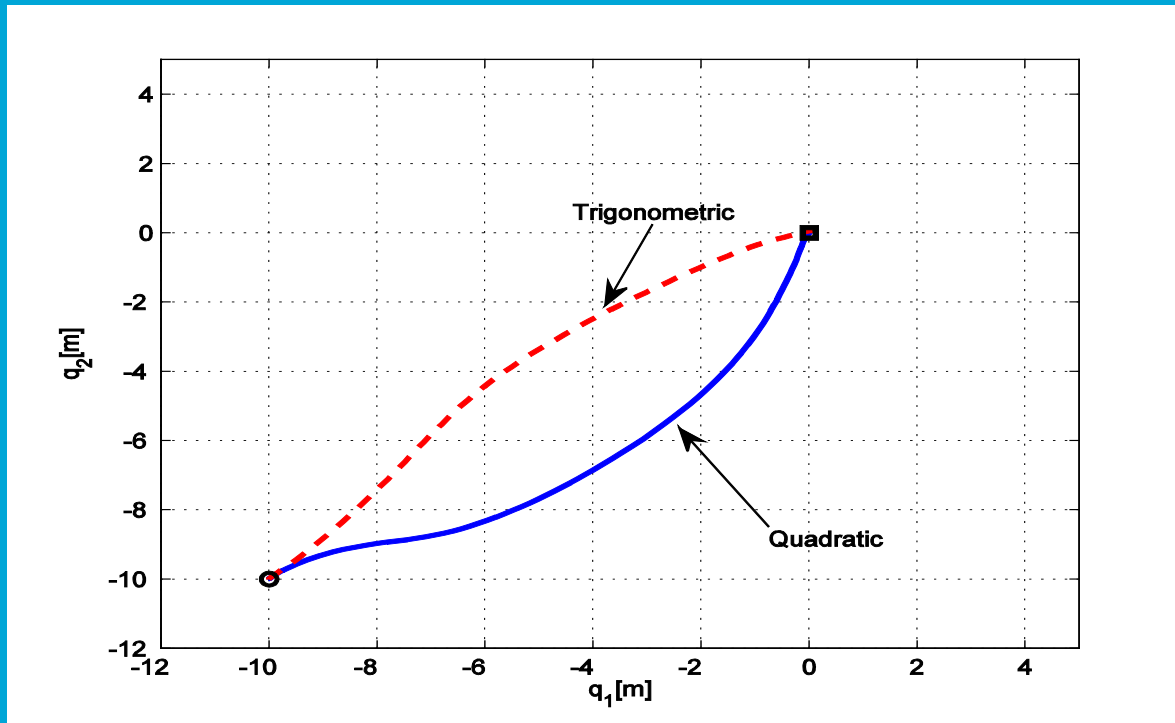


Figure 7. Ship position coordinates in the  $q_1q_2$ -plane, for the static quadratic controller (solid line) and the trigonometric controller (dashed line).

# Family of Passivity Based Controllers

## 3. Dynamic quadratic controller

A quadratic energy shaping

$$H_{de}(q, z, z_e) = \frac{1}{2} q^T K q + \frac{1}{2} z^T M^{-1} z + \frac{1}{2} z_e^T M^{-1} z_e \quad (11)$$

$$\tau_{dq} = -K_p J^T(q_3) K q + K_I z_e - K_D J^T(q_3) \dot{q} - J^T(q_3) b \quad (12)$$

$$\dot{z}_e = -J^T(q_3) K q \quad (13)$$

A nonlinear output feedback PID-control law with a feed forward term

# Family of Passivity Based Controllers

## 4. Dynamic trigonometric controller

A trigonometric energy shaping

$$H_{de}(q, z, z_e) = \frac{1}{2} q_{12}^T C_{12} q_{12} + c_3(1 - \cos q_3) + \frac{1}{2} z^T M^{-1} z + \frac{1}{2} z_e^T M^{-1} z_e \quad (15)$$

$$\tau_{dq} = -K_p J^T(q_3) C \begin{bmatrix} q_1 \\ q_2 \\ \sin q_3 \end{bmatrix} + K_I z_e - K_D J^T(q_3) \dot{q} - J^T(q_3) b \quad (16)$$

$$\dot{z}_e = -J^T(q_3) C \begin{bmatrix} q_1 \\ q_2 \\ \sin q_3 \end{bmatrix} \quad (17)$$

A nonlinear output feedback PID-control law with a feed forward term

# Family of Passivity Based Controllers

- Role of derivative term in control laws

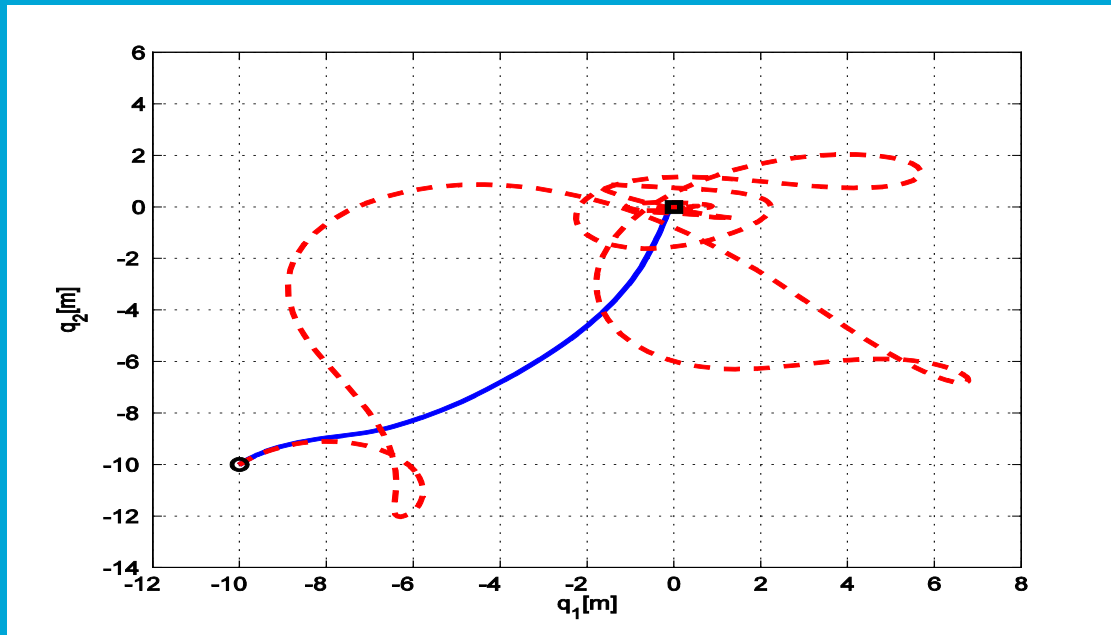


Figure 8. Ship position trajectories in the  $q_1q_2$ -plane, for the static quadratic controller with extra damping (solid line) and the original damping coefficient (dashed line).

# Family of Passivity Based Controllers

- Static vs dynamic controllers

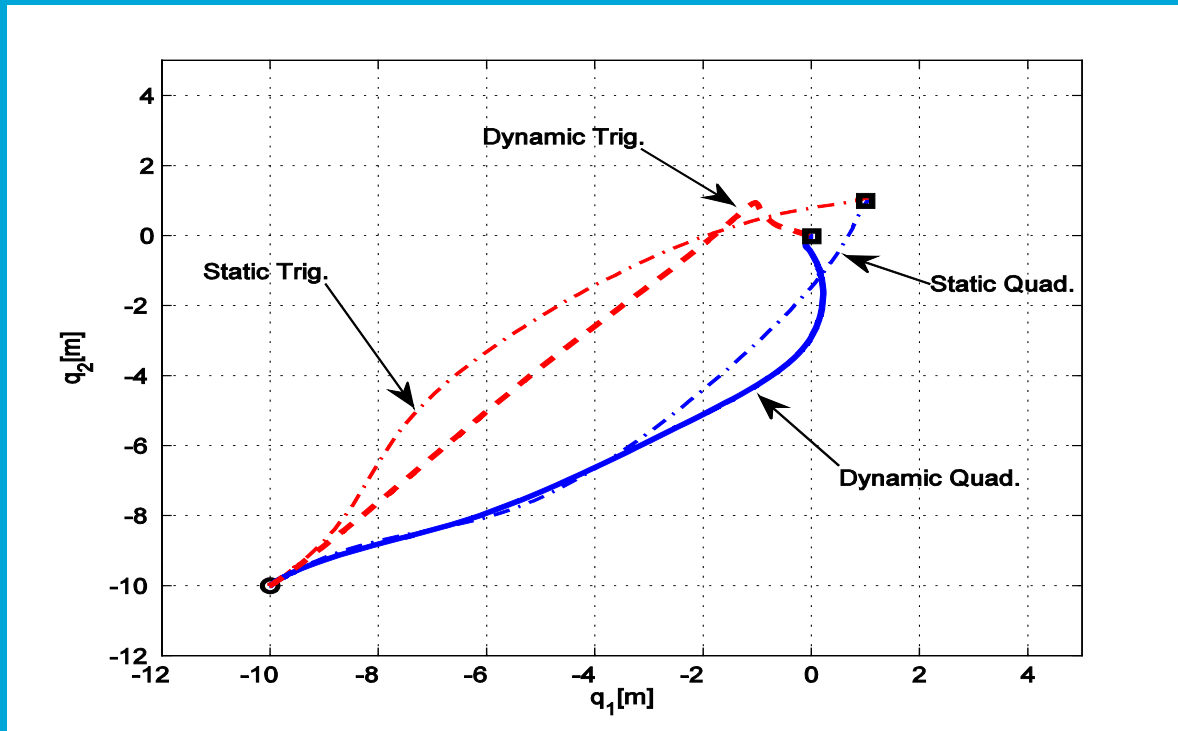


Figure 9. Ship position coordinates in the  $q_1q_2$ -plane, for the static (quadratic and trigonometric) controller (solid line) and the dynamic (quadratic and trigonometric) controller (dashed line).

# Summary and Conclusions

- Port-Hamiltonian methodology and IDA-PBC technique
- Dynamic positioning problem of a sea vessel in port-Hamiltonian structure
- A family of passivity based controllers with simulation results is presented
- Four main observations: (i) The controllers obtained are output feedback (ii) Heading angle can efficiently be controlled by using trigonometric energy shaping (iii) In the presence of disturbances , dynamic controller yields the desired performance (iv) The derivative term in controller equation increases the damping effect and results in a smooth trajectory .

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