



THRUSTERS

Optimal Thrust Allocation in DP Systems

Dr. John Leavitt

L-3 Communications Dynamic Positioning and Control Systems

October 7-8, 2008

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Optimal Thrust Allocation in a DP System

By

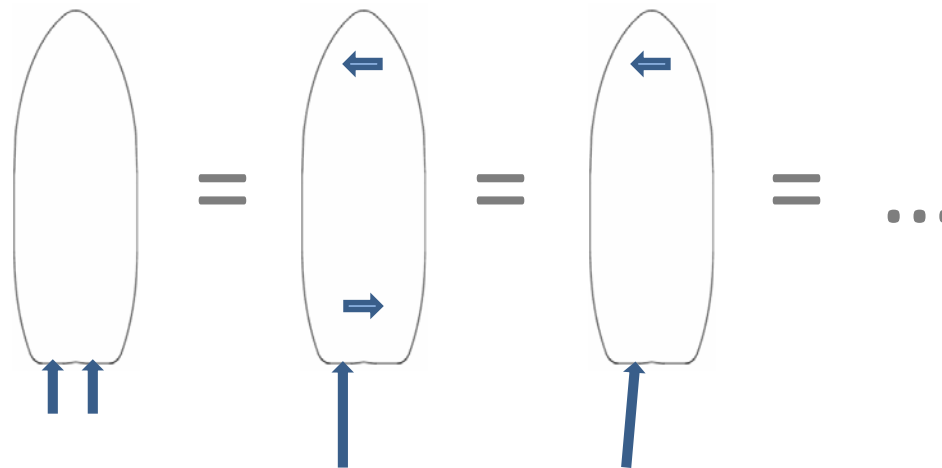
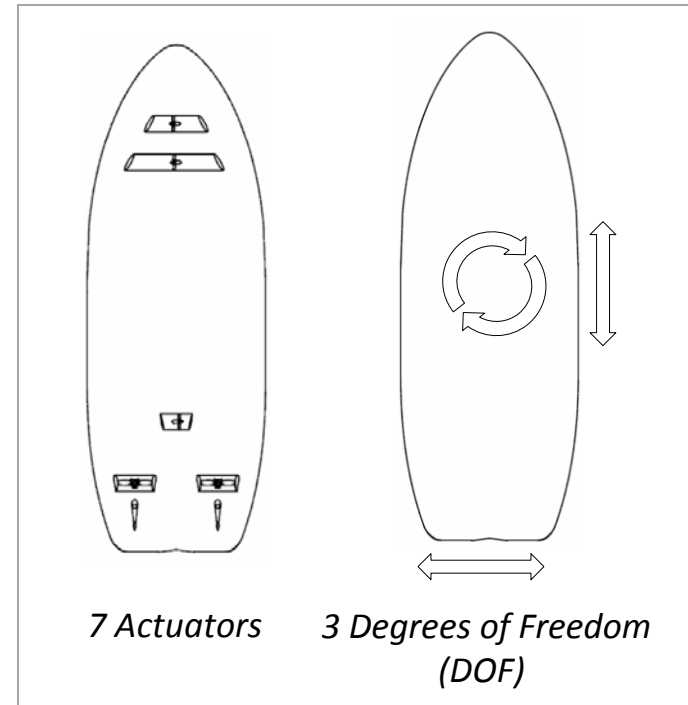
Dr. John Leavitt

L-3 Communications DPCS



INTRODUCTION

- What is thrust allocation?
- With DP, we seek full control of all 3 DOF
- Due to redundancy requirements, most DP vessels are over-actuated
- Example: 7 actuators > 3 DOF = over-actuated
- overactuated = infinite allocation solutions



INTRODUCTION

- What is optimal thrust allocation?
 - the “best” solution with respect to a number of requirements

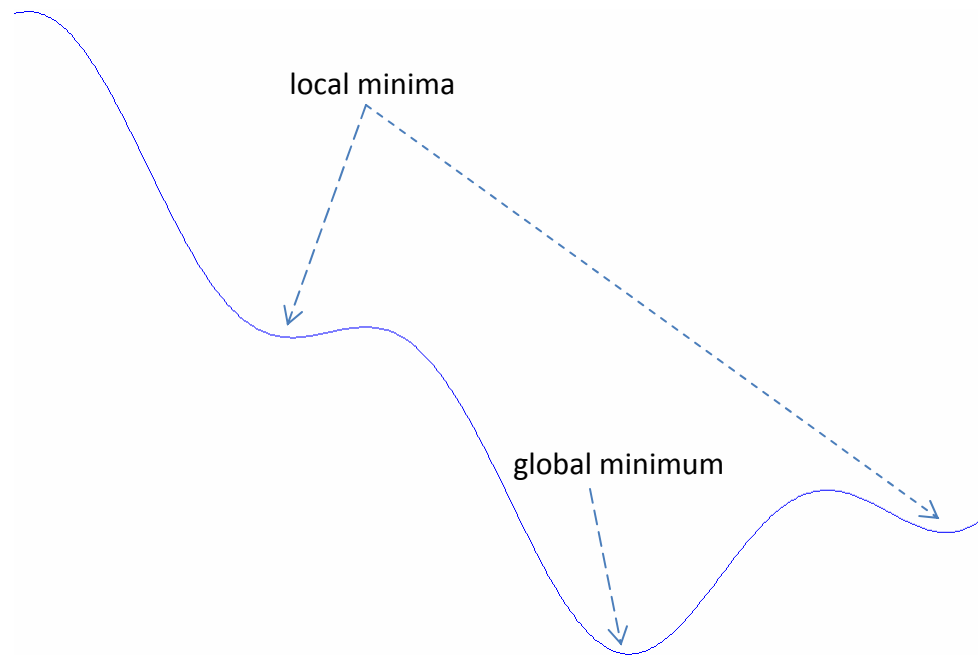
General Requirements:

- 1.Low power
- 2.Low peak thrust levels
- 3.Minimize actuator oscillations (e.g. fluttering rudder)
- 4.Yaw-axis priority

- quantify how poorly an allocation solution meets each requirement
- quantities are weighted and summed, yielding “cost”
- find the solution that minimizes cost, without violating constraints:
 - rate constraints (e.g. slew rate)
 - power limits
 - thrust limits
 - etc.
- problem must be solved once every control cycle!

INTRODUCTION

- quadratic programs, if formulated correctly, can be explicitly and globally solved



THE QUADRATIC PROGRAM

QUADRATIC PROGRAM (QP)

Standard QP

-quadratic cost function J:

$$J(u) = u^T M u + c^T u$$

-linear constraints:

$$G u \leq w$$

-if $M > 0$ (all positive eigenvalues), and J is bounded below, a *unique* global minimum exists

Multi-parametric QP (mp-QP)

-quadratic cost function J, *parameterized* by x

$$J(u) = u^T M u + (x^T F + c^T) u$$

-linear constraints, *parameterized* by x:

$$G u \leq w + E x$$

-parameter space (convex polyhedral) defined by:

$$A x \leq b$$

-if $M > 0$, and J is bounded below, a unique global minimum exists for any x

-explicit solutions w.r.t. x can be calculated

EXPLICIT SOLUTIONS TO MP-QP

- CR is the parameter space defined by:

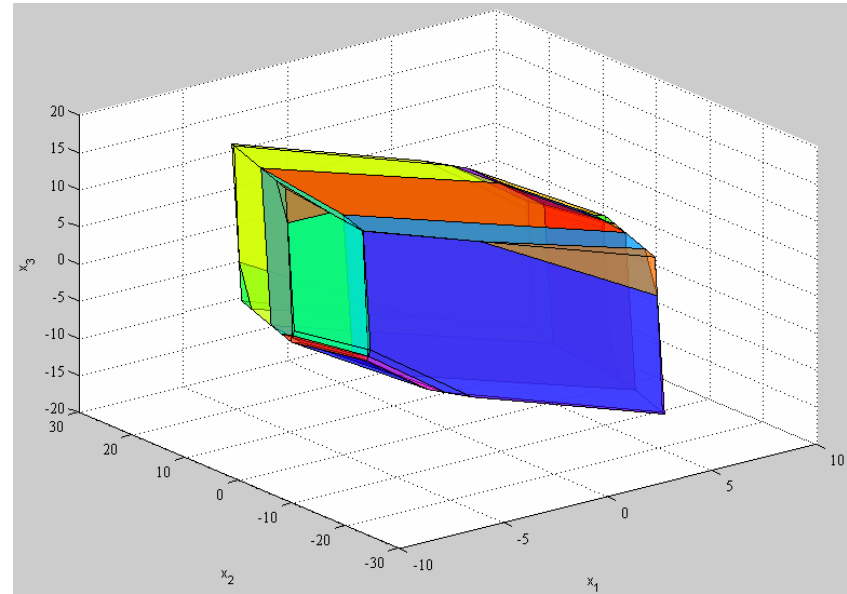
$$CR = \{x : Ax \leq b\} = \cup_{i=1}^N CR_i$$

- where the subspace

$$CR_i = \{x : H^i x \leq k^i\}$$

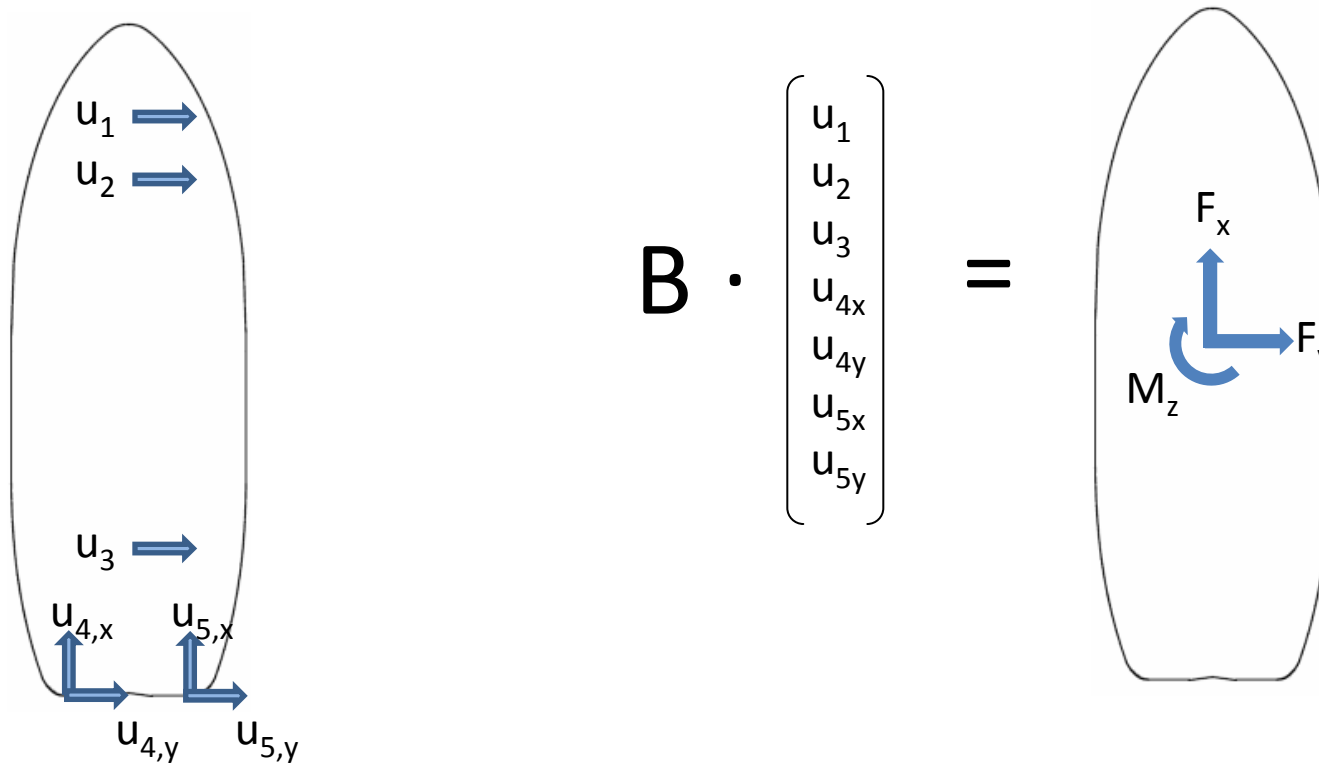
- to implement the solution for a given x , search to find which subspace x belongs to, then calculate:

$$u_* = F^i x + g^i$$



JOHANSEN'S ALLOCATION FORMULATION

- JOHANSEN, T. A., T.P. FUGLSETH, P. TØNDEL, and T.I. FOSSEN. "Optimal Constrained Control Allocation in Marine Surface Vessels with Rudders." *IFAC 6th Conference on Maneuvering and Control of Marine Crafts*, IFAC, 2003, pp. 215–220.



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- u , "extended thrust" vector

- B , allocation matrix, transforms individual actuator commands to net force and moment

- s , "slack vector" that allows for prioritized cutbacks $s = \tau_c - Bu$

- Cost Function:

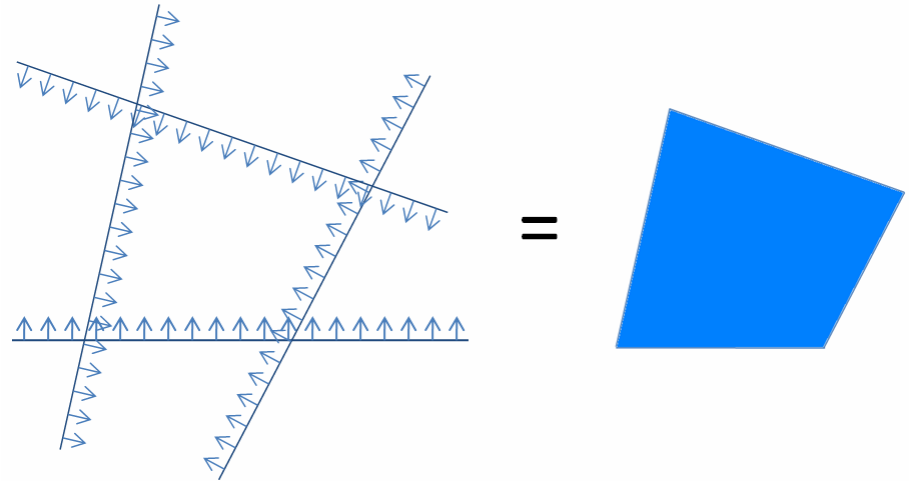
$$J(u, x) \equiv u^T H u + s^T Q s$$

LINEAR CONSTRAINTS, $Ax \leq b$

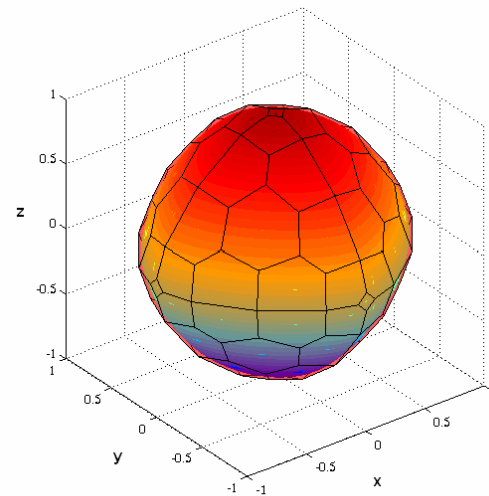
- the above statement is a compact way of writing:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m \end{cases}$$

- Example: when $n = 2$,
we have a convex polygon



- Example: when $n = 3$,
we have a convex polyhedral
- m is the number of edges, facets, or *hyper-planes* that define the region



ALLOCATION CONSTRAINTS

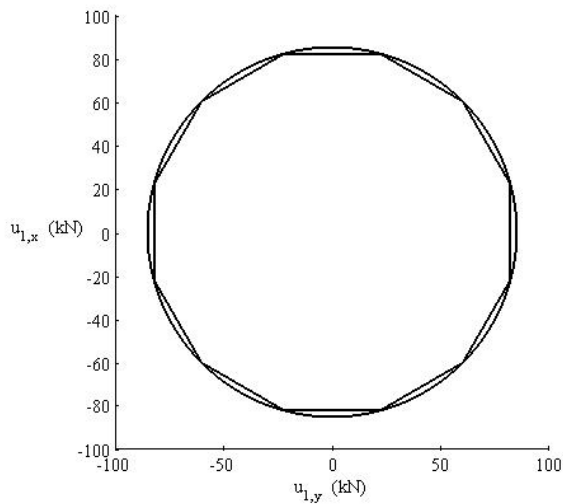
- thrust limits:

$$r^2 = u_x^2 + u_y^2 \leq r_{\max}^2$$

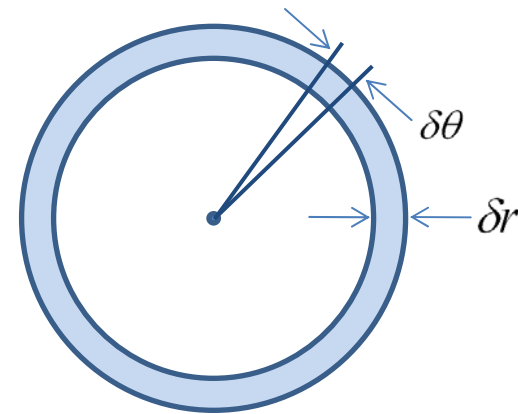
- rate limits:

$$\dot{\theta} \leq \dot{\theta}_{\max}, \quad \dot{r} \leq \dot{r}_{\max}$$

- power limits can in some cases be *linearized* as well

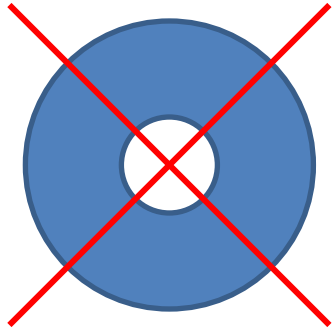


thrust limits for an azimuthing thruster

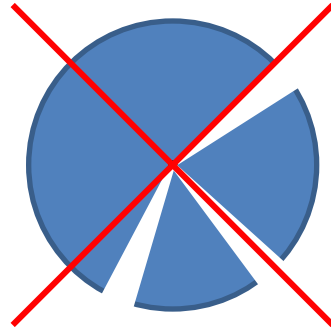


rate limits for an azimuthing thruster

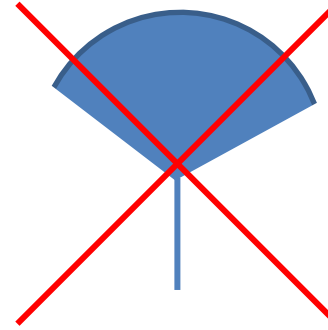
LIMITATIONS OF $Ax \leq b$



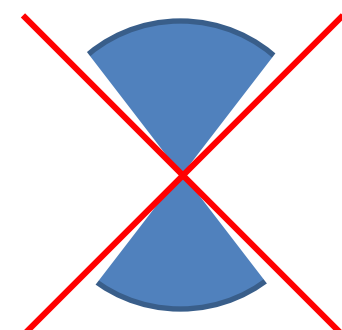
Non-zero Minimum Thrust
(e.g. clutched engine)



Spoil Zones



Rudders



Bidirectional Azis w/
Limited Range of
Motion

- Power limits can only be handled in certain cases

Workarounds

- Rudder thrust regions can be split into two convex regions (see Johansen)
- Spoil zones and bidirectional thrusters can be handled likewise
- Spoil zones aren't configurable when handled this way, and offline computation times are potentially a nightmare
- Non-zero minimum thrust **cannot** be handled in QP framework

LIMITATIONS OF A QUADRATIC COST FUNCTION

- neither total power nor peak thrust levels are explicitly minimized (not a real issue).
- no cost can be assigned to rotational motion
RESULT: flutter, oscillation, hunting, etc.
WORKAROUND: Johansen's shifting method, rudders only!

SUMMARY OF MP-QP WEAKNESSES

- azimuthing thrusters will rotate excessively in low sea states
- spoil zones cannot effectively be handled
- non-zero minimum thrust cannot effectively be handled for azimuthing thrusters
- explicit rate limits for azis are not possible
- power limiting can only be handled for simple cases, but with prohibitive offline computation times in a commissioning setting

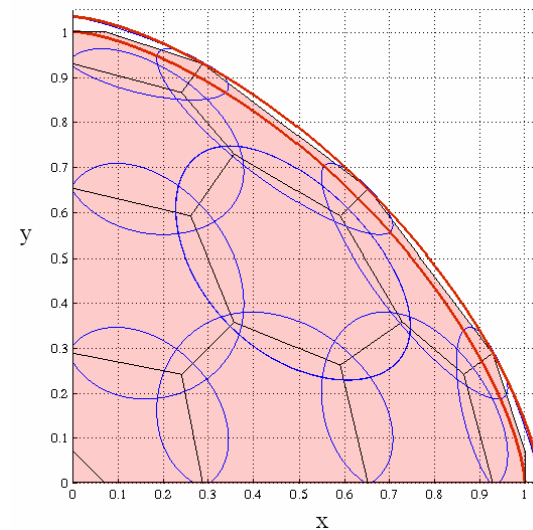
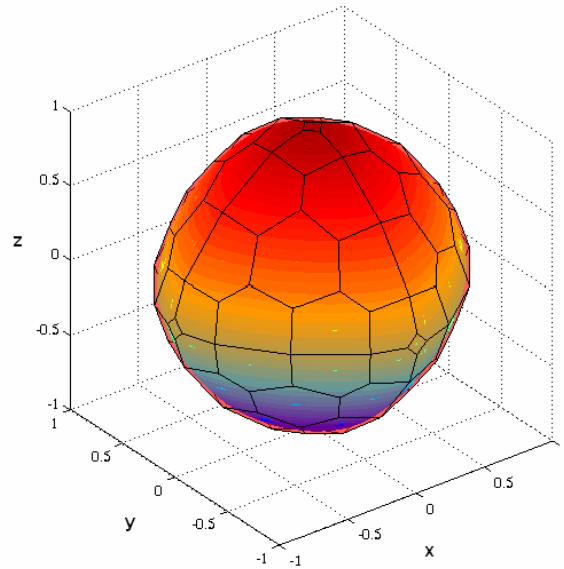
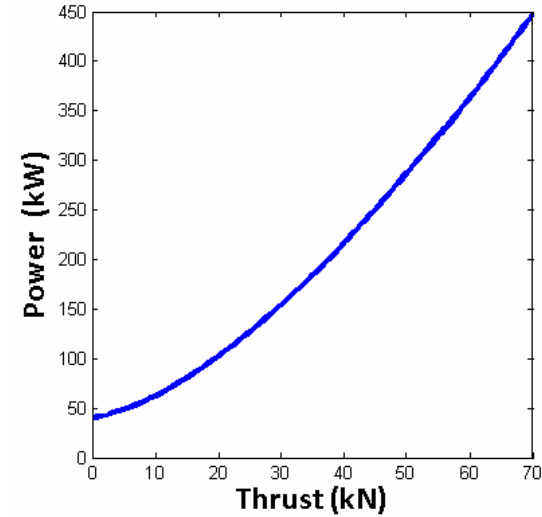
EXAMPLE OF POWER LIMITING USING MP-QP

- Approximating power from thrust:

$$P = (P_{\max} - P_{\min}) \left(|T| / T_{\max} \right)^\eta + P_{\min}$$

- Example: three thrusters on a power bus

$$\frac{P_{1,\max}}{T_{1,\max}^{1.5}} |T_1|^{1.5} + \frac{P_{2,\max}}{T_{2,\max}^{1.5}} |T_2|^{1.5} + \frac{P_{3,\max}}{T_{3,\max}^{1.5}} |T_3|^{1.5} \leq \Psi$$

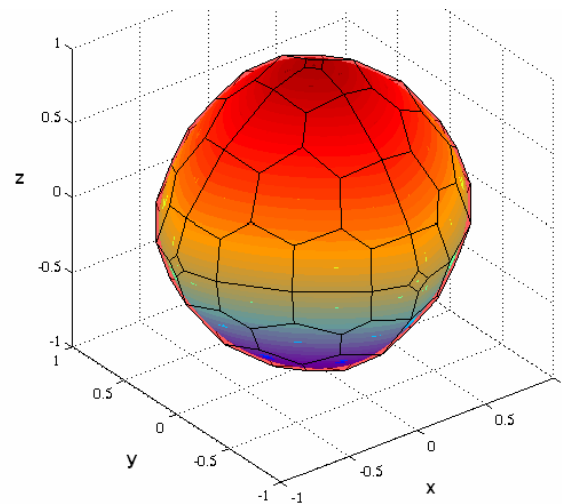


POWER LIMITING USING MP-QP

- Power limits Ψ can be added to the parameter list x
- Polyhedral scales easily using parameterized constraints:
$$Gu \leq w + Ex$$

DRAWBACKS

- Offline computation times are prohibitive for even simple cases
- Linearization procedure is difficult for higher dimensions

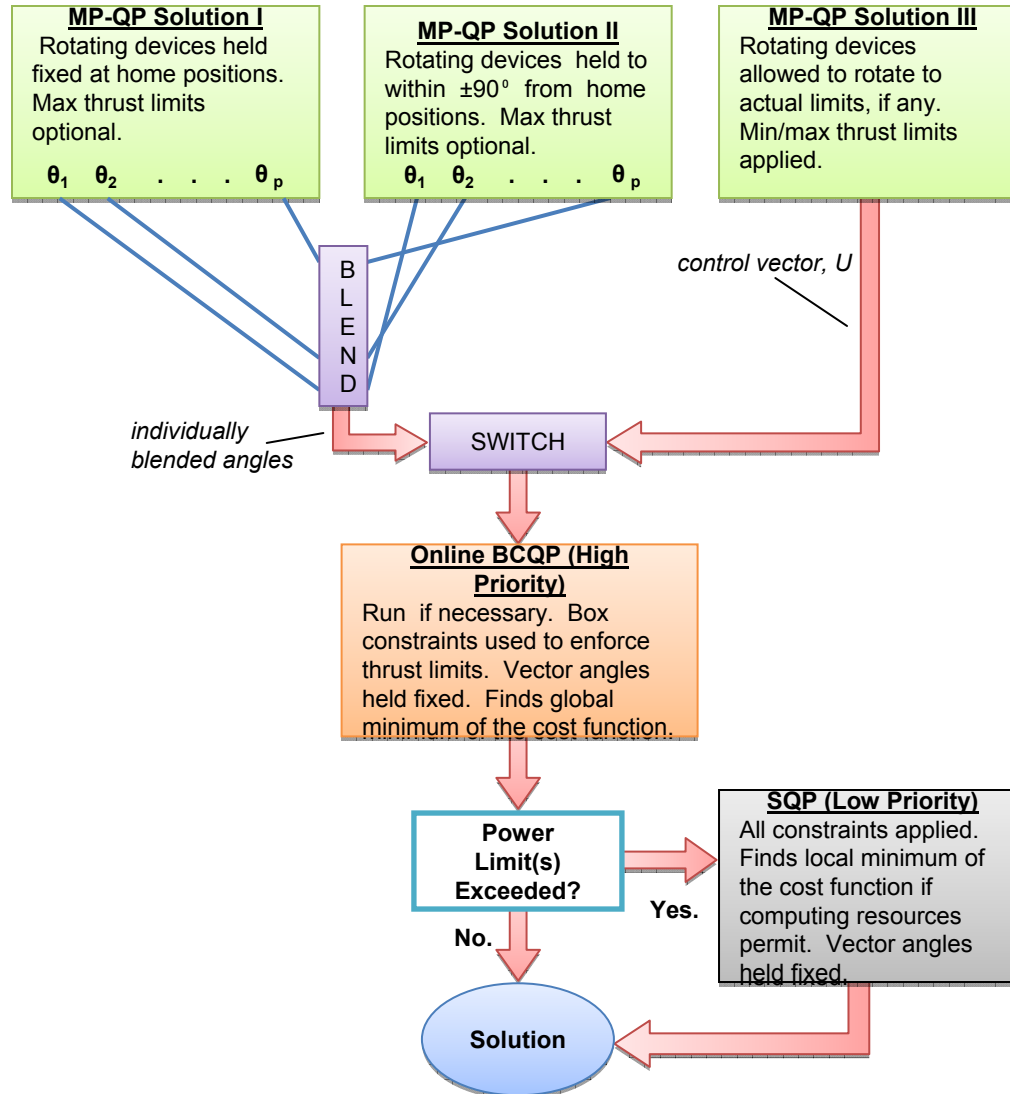


PROPOSED SOLUTION

GENERALIZED ALLOCATION ALGORITHM

- rotating devices are handled by blending 3 MP-QP solutions
- Solution I – fixed axis
- Solution II – rotation is semi-constrained
- Solution III – rotation is not artificially constrained
- angular blending is individually applied to each device
- algorithm is a hybrid of offline and online approaches

GENERALIZED ALLOCATION ALGORITHM

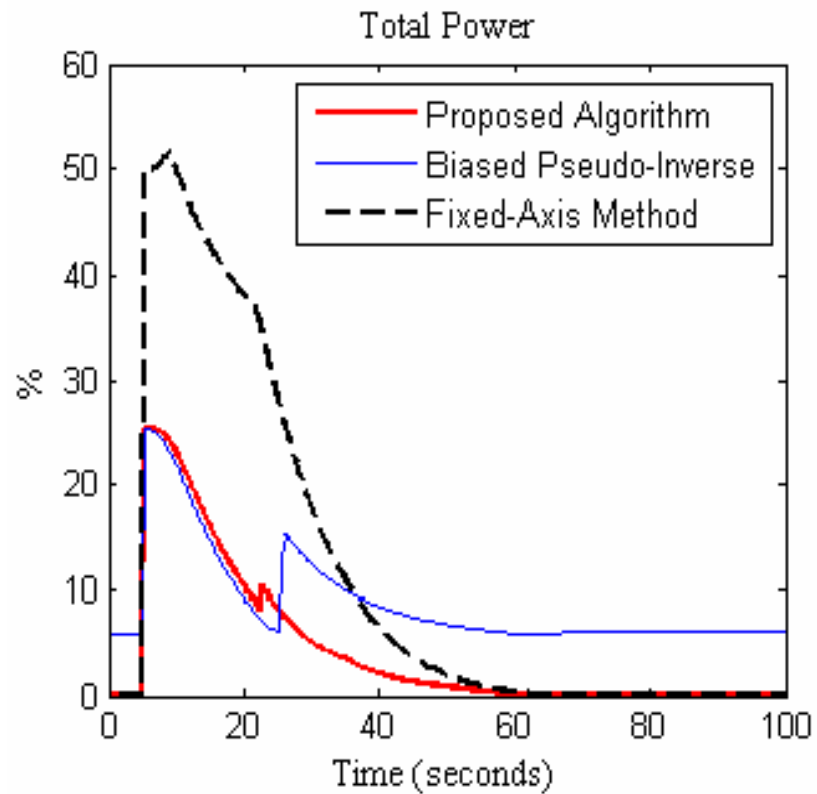


IMPROVEMENTS OVER MP-QP

- hunting/fluttering of rotating devices is operator adjustable
- angular rate limits can be directly applied
- azi home positions are configurable online
- spoil zones, non-zero minimum thrust, etc. can be handled (sub-optimally)
- provision is made for optimal power limiting

MATLAB SIMULATION

- Drillship with 6 azimuthing thrusters
- Random periodic impulse applied
- No power or rate limiting



CONCLUSIONS

- An optimal allocation algorithm has been proposed
- Although it is an online implementation, it evaluates relatively quickly
- The algorithm is flexible enough to be applied to any vessel
- A number of special cases can be handled (power limiting, spoil zones, etc.)