Design and Control

Exponentially Stable Underactuated Dynamic Positioning of Marine Vehicle

Matthew Greytak
Massachusetts Institute of Technology

October 9-10, 2007

Return to Session Directory

Return to Session Directory

Exponentially Stable Underactuated Dynamic Positioning

Matt Greytak
Massachusetts Institute of Technology

Research Sponsored by the Office of Naval Research
Electric Ship Research & Development Consortium

Massachusetts Institute of Technology

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

Return to Session Directory

Outline

- Motivations: Why we’re interested
- Technical Challenges: Why it’s hard
- State Feedback Controller: Partial solution
- Manifold Convergence Controller: The rest
- Simulation Results: Perfection
- Experimental Results: Reality
- Conclusions

Return to Session Directory

General Dynamic Positioning

- Requirements:
  - Many thrusters
  - Thrust allocation algorithm

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

Images: dosecc.org

Underactuated DP

- Fully actuated ship: can generate an acceleration in any direction (surge, sway, yaw)
  - Need at least three independent inputs: 1 azimuth thruster + 1 bow thruster for example
- Underactuated ship: only two inputs
  - 1 azimuth thruster
  - Fixed shaft + rudder
  - Outboard engine?
  - Small craft!

Applications

- Stationkeeping autopilots for fishing vessels and rescue boats
- DP for DP vessels damaged beyond redundancy
- ASV/AUV positioning

Return to Session Directory

Challenge of Underactuation

- Can't reject most disturbances with only two inputs!
  - Fore-aft errors OK, transverse/rotation errors not
- General approaches to underactuated control:
  - Trajectory planning
  - Energy Methods
  - Switched Control
- Planning = High-order Nonlinear Optimization:
  - Computationally intensive
  - Generally open-loop

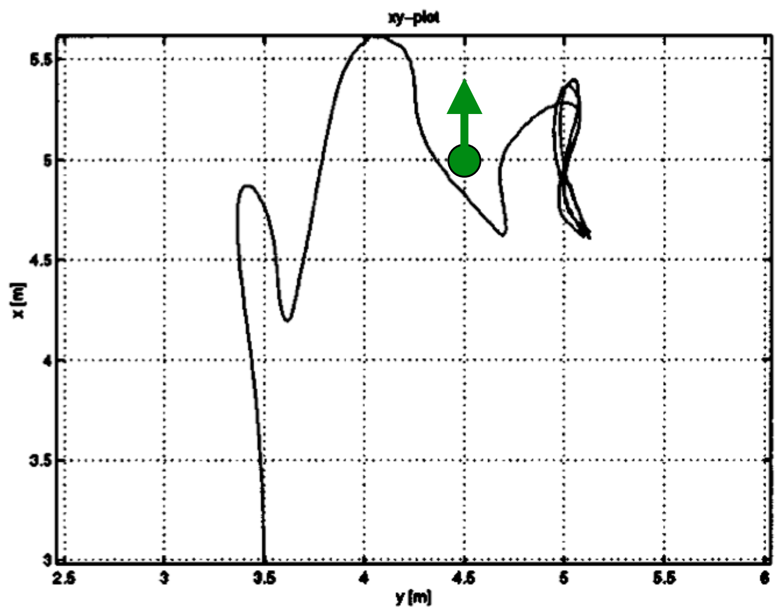
Mathematically…

$$\dot{\nu} = a(\nu)\,\nu + b\,\tau$$

accel 3x1 — velocity 3x1 — input 2x1

- Matrix b is not full rank, so underactuated
- But time-varying periodic control laws can work
- Parallel parking a car

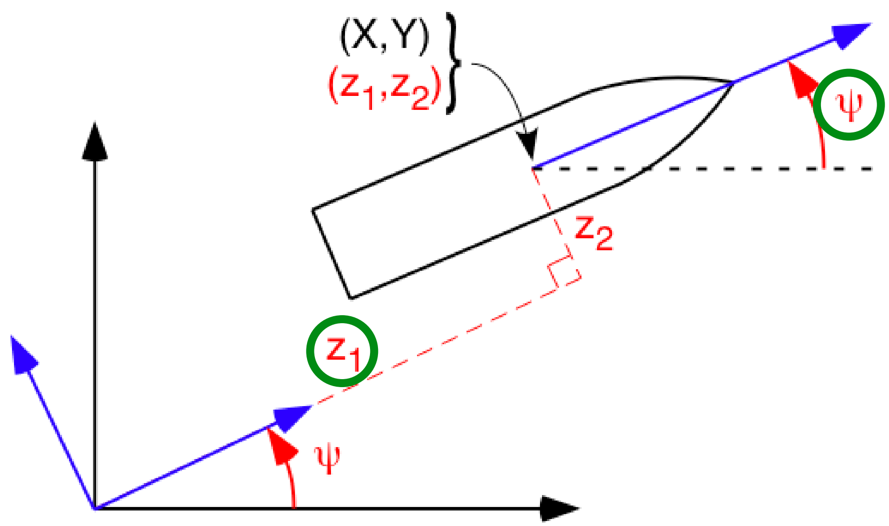
- Pettersen 1996:
- Experiments 2000:



Return to Session Directory

Well what CAN we do?

- With 2 inputs, we can potentially regulate 2 outputs with state feedback
- Linear Quadratic Regulator (LQR) or other…



$$
\begin{aligned}
z_1 &= X\cos\psi + Y\sin\psi \\
z_2 &= -X\sin\psi + Y\cos\psi \\
z_3 &= \psi
\end{aligned}
\qquad
\begin{aligned}
\dot{z}_1 &= u + z_2 r \\
\dot{z}_2 &= v - z_1 r
\end{aligned}
$$

Return to Session Directory

Linear State Feedback

- State transition (linear damping, nonlinear kinematics):

$$\mathbf{x} = [u, v, r, z_1, z_2, \psi]^T \equiv [\boldsymbol{\nu}^T, \mathbf{z}^T]^T$$

$$\begin{aligned} \dot{\mathbf{x}} &= \begin{bmatrix} \mathbf{a}(\boldsymbol{\nu})\boldsymbol{\nu} \\ \dot{\mathbf{z}} \end{bmatrix} + \begin{bmatrix} \mathbf{b} \\ \mathbf{0} \end{bmatrix} \boldsymbol{\tau} \\ &\equiv \mathbf{f}(\mathbf{x}) + \mathbf{B}\boldsymbol{\tau} \end{aligned} \qquad \mathbf{f}(\mathbf{x}) = \begin{bmatrix} a_{11}u + \frac{m_{22}}{m_{11}}vr \\ a_{22}v + a_{23}r - \frac{m_{11}}{m_{22}}ur \\ \frac{m_{11}-m_{22}}{m_{33}}uv + a_{32}v + a_{33}r \\ u + z_2 r \\ v - z_1 r \\ r \end{bmatrix}$$

every state except z_2 (transverse error)

- LQR controller:

$$J = \int_0^\infty \mathbf{x}^T \mathbf{Q}_5 \mathbf{x} + \boldsymbol{\tau}^T \mathbf{R} \boldsymbol{\tau} dt$$

State Feedback Control Law

$$\begin{aligned} \boldsymbol{\tau} &= -\mathbf{R}^{-1}\mathbf{B}^T\mathbf{C}_5^T\mathbf{P}_5\mathbf{C}_5\mathbf{x} \\ &\equiv -\mathbf{K}\mathbf{x} \end{aligned}$$

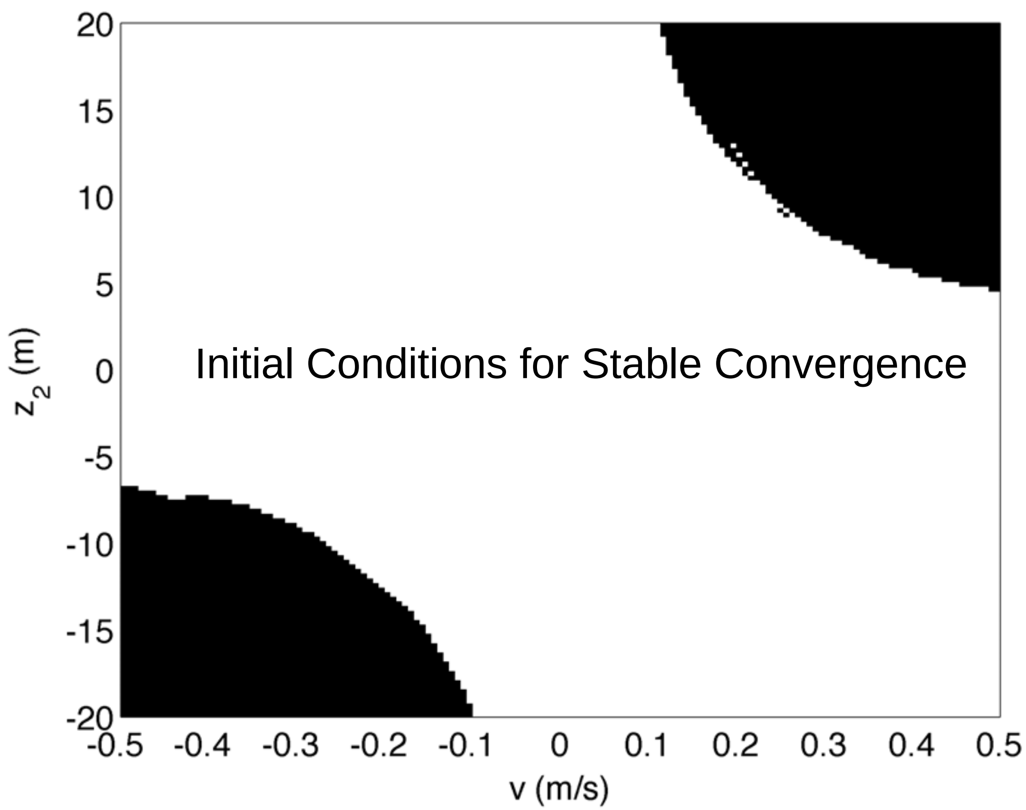
Closed Loop Dynamics

$$\begin{aligned} \dot{\mathbf{x}} &= (\mathbf{f}_{\mathbf{x}} - \mathbf{B}\mathbf{K})\mathbf{x} \\ &= \mathbf{f}_{\mathbf{x}}^{cl}\mathbf{x} \end{aligned}$$

Return to Session Directory

Region of Convergence

- Strong z_2v dependence, weak u dependence
- Region of convergence large in this example (LOA = 3.71 m)



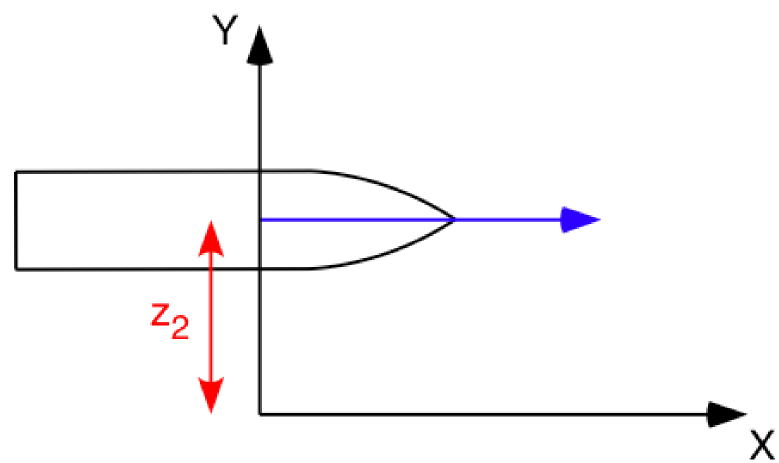
Return to Session Directory

Great… now what?

- The LQR (or other) state feedback controller will drive all states except z_2 to zero

$$\begin{matrix} & u & v & r & z_1 & z_2 & \psi \\ \mathbf{x}(\infty) = [& 0 & 0 & 0 & 0 & z_2(\infty) & 0 &]^T \end{matrix}$$

- Vessel ends up on the global Y axis pointing along the X axis

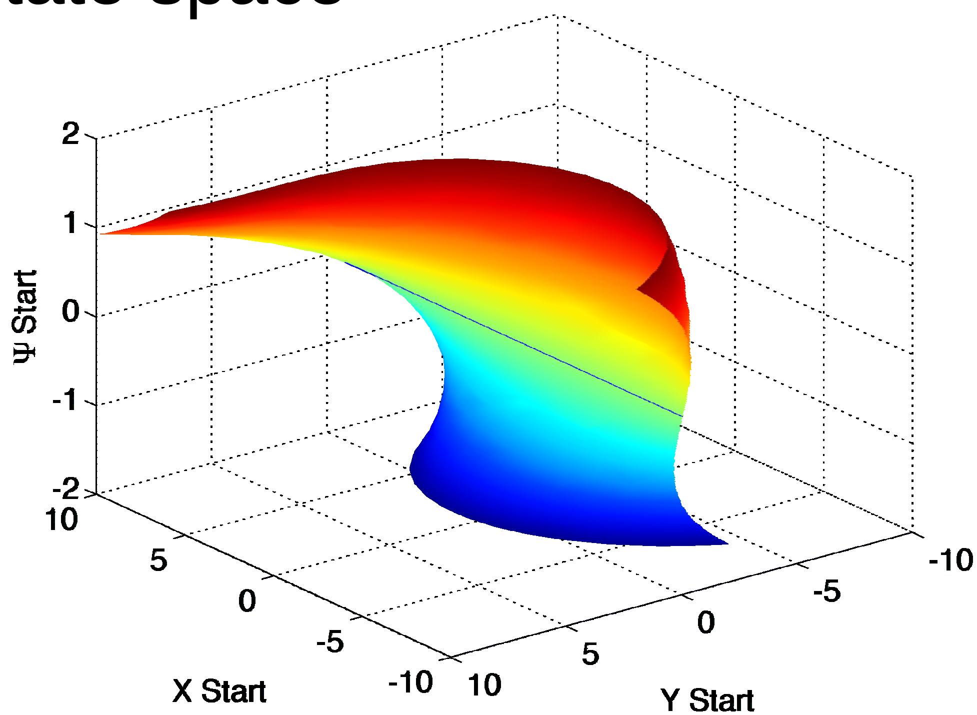


- With a simple simulation we can predict $z_2(\infty)$ from any initial condition!

Return to Session Directory

The Manifold

- For certain initial conditions the final (time infinity) state vector will be zero
- This set of initial conditions forms a manifold in the state space



Return to Session Directory

Algorithm

1. Get to the manifold — HARD?
2. Drive along the manifold to the goal — EASY!
3. Get back to the manifold if you get pushed off

Intuition (behavior we want)

- For small disturbances, just use the state feedback controller and accept minor errors
- For larger disturbances, you need to reposition yourself… may need to back off from the goal for a moment (parallel parking analogy)

Return to Session Directory

Manifold Convergence Controller

- Find a positive definite function of some states of interest; if its derivative with respect to time is negative definite then those states will go to zero

$$V = \frac{1}{2} {z_{2\infty}}^2$$

predicted final lateral offset

$$\begin{aligned} \dot{V} &= \frac{\partial V}{\partial \mathbf{x}} \frac{d\mathbf{x}}{dt} \\ &= \frac{\partial V}{\partial \mathbf{x}} \left((\mathbf{f_x} - \mathbf{BK})\, \mathbf{x} + \mathbf{B}\, \boldsymbol{\tau_l} \right) \\ &= z_{2\infty} \frac{\partial z_{2\infty}}{\partial \mathbf{x}} (\mathbf{f_x} - \mathbf{BK})\, \mathbf{x} + z_{2\infty} \frac{\partial z_{2\infty}}{\partial \mathbf{x}} \mathbf{B}\, \boldsymbol{\tau_l} = c_v \end{aligned}$$

control input

negative definite

$$-V/T_V$$

Manifold Convergence Controller (MCC)

$$\boldsymbol{\tau_l} = -pinv\left(\frac{\partial z_{2\infty}}{\partial \mathbf{x}} \mathbf{B} \right) \left(\frac{1}{2T_V} z_{2\infty} + \frac{\partial z_{2\infty}}{\partial \mathbf{x}} (\mathbf{f_x} - \mathbf{BK}) \mathbf{x} \right)$$

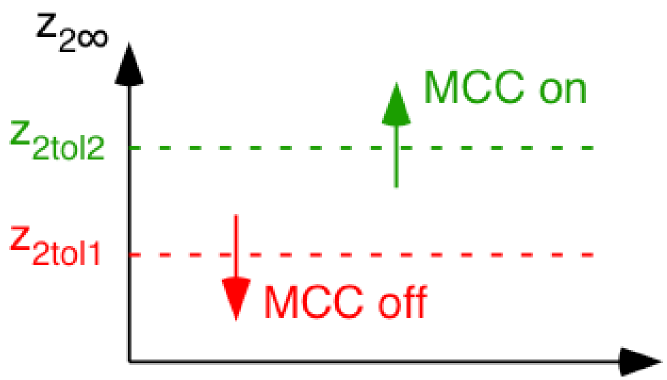
numerical derivative

state feedback!

Return to Session Directory

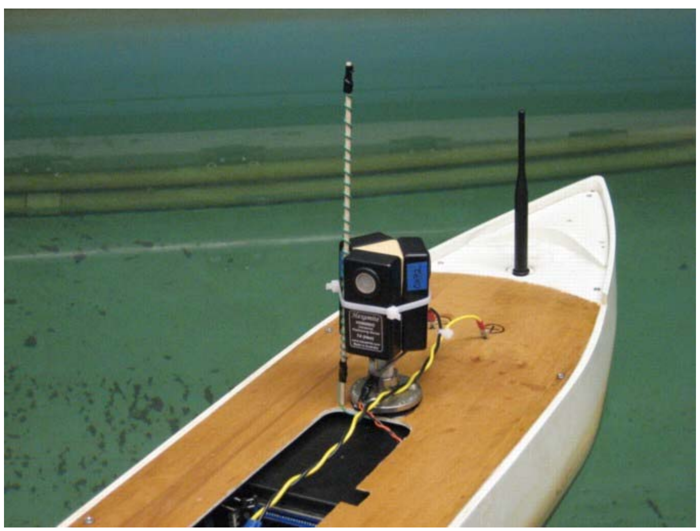
Wait a sec!

- We're controlling a future state with current inputs… that could be problematic!
- … and it is! The MCC gets to the manifold asymptotically, but not the goal.
- When within some tolerance around the manifold, switch to the state feedback controller which drives toward the goal.
- Use a switching boundary with hysteresis:
  - Turn off MCC when inside a certain radius, turn it back on if you get outside a larger radius



Return to Session Directory

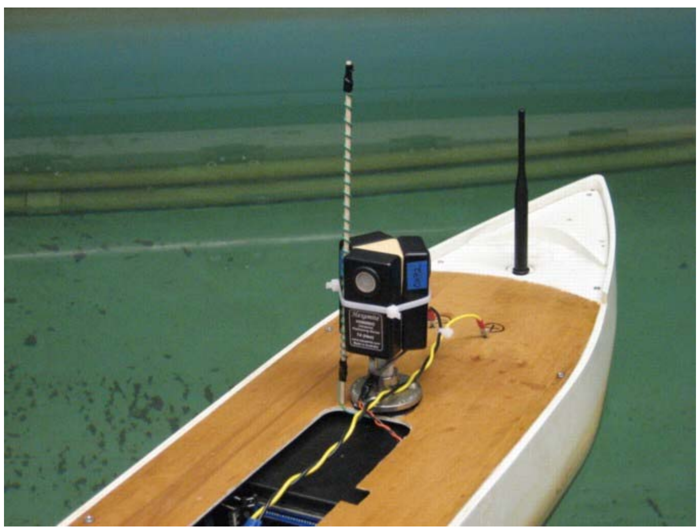
Simulation of Icebreaker Model

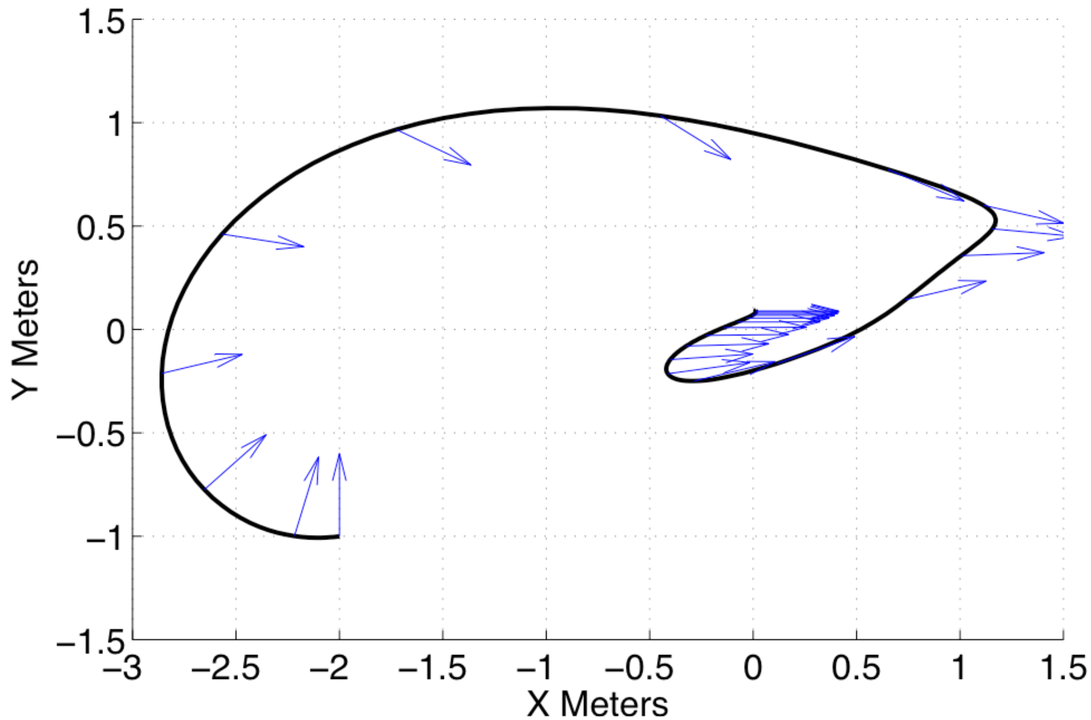


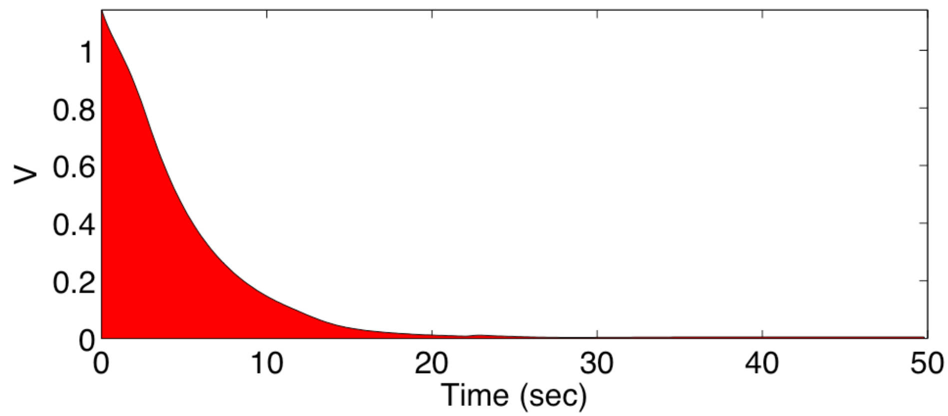
QuickTime™ and a
Cinepak decompressor
are needed to see this picture.

Return to Session Directory

Simulation Results







Return to Session Directory

Correct for Modeling Error (experiment)

QuickTime™ and a
Cinepak decompressor
are needed to see this picture.

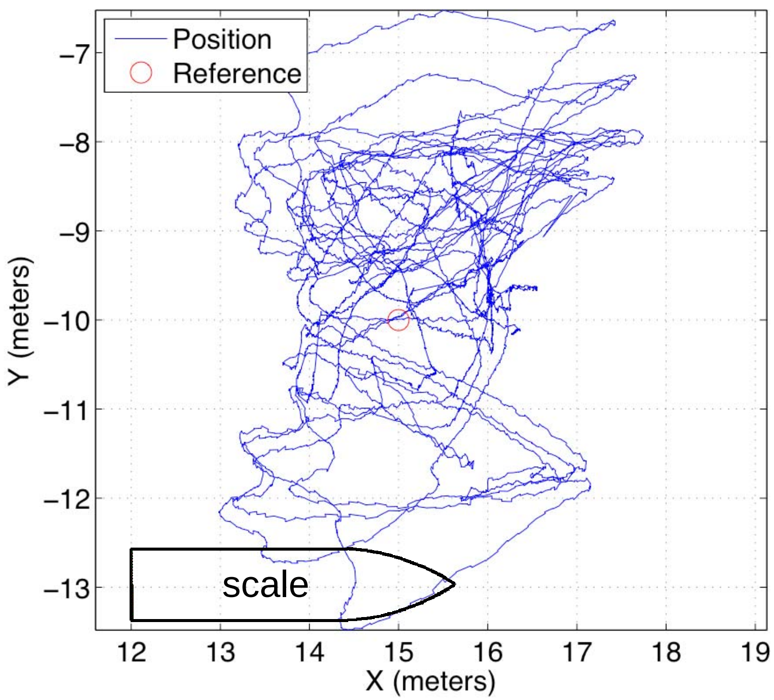
Return to Session Directory

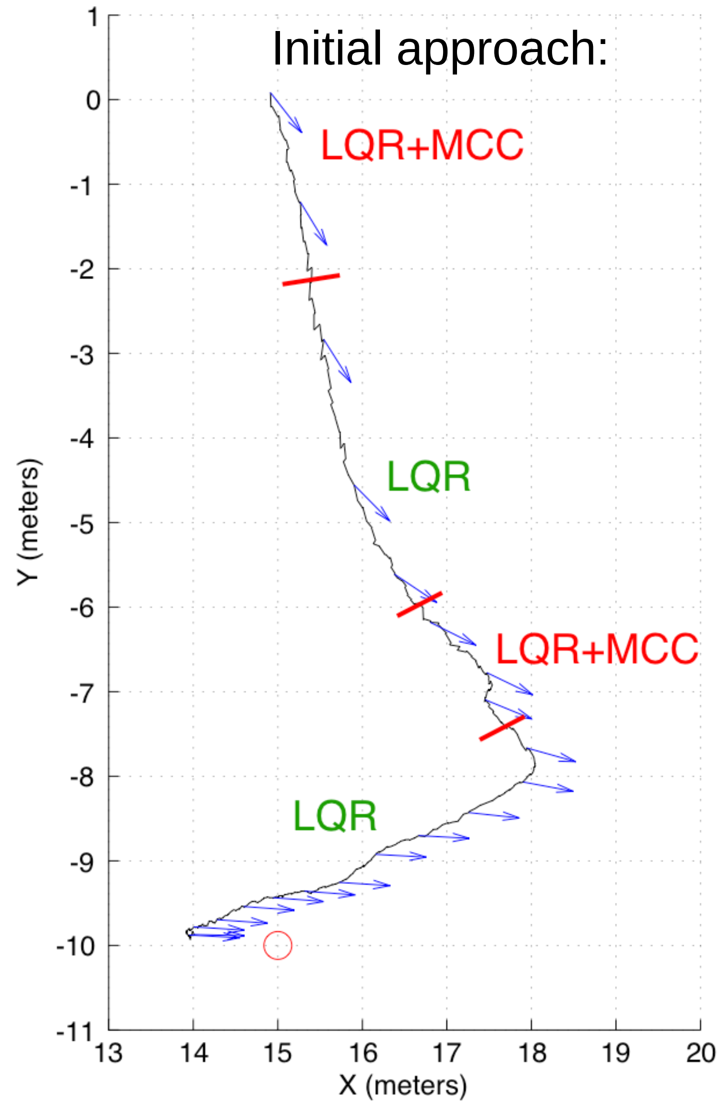
On the Charles River (experiment)

QuickTime™ and a
Cinepak decompressor
are needed to see this picture.

Return to Session Directory

On the Charles River (experiment)





Return to Session Directory

Conclusions

- Underactuated DP is possible if the vessel is allowed to reposition itself when errors get too large
  - Minimize future errors with current actions
  - Exponentially stable switching control law
  - Intuitive results
- Issues:
  - Need a very good model of the vessel and, if possible, environmental conditions (wind & current)
  - Errors not driven exactly to zero

Return to Session Directory

What's Next?

- Use a better (nonlinear) model in the forward simulations
- Include known environmental forces
- Extend to 5 DOF with 3 input channels (AUV with propeller, rudder & dive planes)

Return to Session Directory

Questions?